QCD phase transition in rotaing neutron star, Neutrino beaming and Gamma-ray bursters

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We have studied the emission of neutrinos from a rotating hybrid star. We find that the emission is predominantly confined to a very small angle, provided the core of the star is in a mixed phase of quarks and hadrons and the size of such a mixed phase is small. Annihilation of neutrinos to produce gamma rays has been discussed. The estimated duration of the burst is found to be within the observational range.

The existence of strange quark matter (SQM) containing u, d and s quark had been postulated quite some time ago. It was also proposed that the SQM could be the *true ground state* of quantum chromodynamics [1]. This conjecture has been supported by various model calculations for certain ranges of the model parameters [2,3]. If this is so, then the usual hadronic matter could undergo a phase transition to SQM at high temperature and/or density. This opens up the possibility that the interior of neutron stars may consist of SQM, the baryon density there being extremely large (8-10 times that of nuclear matter density at saturation).

In SQM, the strangeness fraction *i.e.* the ratio of strange quark and baryon number densities, will be unity, if one considers the masses of u, d and s to be same. Even for realistic strange quark masses ($m_s > m_u = m_d$), the strangeness fraction in quark matter at high temperature/density is not much smaller than unity. In contrast to quark matter, the strangeness fraction in hadronic matter is usually found to be small [4–6]. Sizable strangeness fraction can be accommodated in hadronic models by including hyperons and/or kaon condensation [7]. For the mean field models with hyperons, the strangeness fraction has been shown to be smaller than unity [6], at densities where the transition to quark matter may occur. Depending on the model parameters and the interactions considered, the situation may be different for an equation of state (EOS) with kaon condensation [7,8]. But for such cases, the transition to quark matter is found to be pushed towards much higher densities.

The above discussion implies that the transition from hadronic matter to SQM inside neutron stars may be associated with a large production of strangeness. This can also be explained in the following way. Initially, hadronic matter, in terms of quark content, consists predominantly of u and d quarks and possibly a very small fraction of s quarks (from hyperons). The quark matter thus formed through a phase transition from the hadronic matter should be out of chemical equilibrium. The weak interactions then convert this chemically non-equilibrated matter to equilibrated SQM with roughly equal numbers of u, d and s quarks. This conversion is associated with the production of large amounts of energy in the form of neutrinos, with average energy of the order of 100 MeV [9].

The observed Gamma Ray Bursters (GRBs) [10] continue to be a puzzling astrophysical phenomenon, insofar as there does not exist an universally accepted explanation as yet. In recent times, however, it has been argued that most GRBs are associated with supernovae [10] and thus, efforts are on to arrive at some picture for the GRB engine consistent with the supernova explosion. In ref. [11], Berezhiani *et al.* have suggested that the origin of the GRBs may be associated with the deconfinement transition inside a neutron star resulting into a hybrid star (neutron star

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with quark matter in its centre, either in a mixed phase or in a pure quark matter state) or a pure quark star. In their model, the time delay between the supernova explosion which creates the neutron star and the GRB is governed by matter accretion on the neutron star; only when sufficient matter has been accreted does the neutron star core density cross the threshold for the phase transition to quark matter. While this picture provides a plausible scenario for the GRB engine and explains the time delay between the supernova explosion and the GRB, several features of GRB, namely the amount of energy release, the duration of the burst and most importantly, the observed beaming [12], remain unaddressed.

In the present letter, we extend the basic premise of ref. [11] to rotating stars. As a rotating neutron star gets converted into a hybrid star, with the same total baryonic number, the deconfinement transition is accompanied by the conversion of predominantly two flavour matter into strange quark matter through weak interaction. As a result, a large number of neutrinos may be produced during the phase transition from hadronic matter to strange quark matter. The annihilation of neutrinos [13] thus produced gives rise to gamma ray bursts. Hence, we also estimate the duration of burst from the time span of neutrino production.

In the following we proceed as follows. We first construct the full EOS for the chemically equilibrated hadronic matter with phase transition to quark matter. This EOS gives us the final composition. *i.e.*, the fraction of baryons and quarks in the final star in equilibrium. This in turn decides how much of the baryons are finally dissolved to quark matter. Once we know these fractions, we can now use the formalism of ref. [9] to estimate the total energy, number of neutrino and time taken for the chemical equlibration during the conversion.

The full EOS of the neutron star with a quark core is constructed using two different models for hadronic and quark sector. The mixed phase is obtained following Gibbs criteria. The hadronic part of the EOS has been constructed using the TM1 parameter set of the non-linear Walecka model. The corresponding lagrangian density is [14]:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{YY} + \mathcal{L}_l \tag{1}$$

where

$$\begin{aligned}\mathcal{L}_0 = \sum_B \overline{\psi}_B \left(i\partial\!\!\!/ - m_B\right)\psi_B + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - U(\sigma) - \frac{1}{4}G^{\mu\nu}G_{\mu\nu} + U(\omega) - \frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}^\mu\vec{R}_\mu \\ - \sum_B \overline{\psi}_B \left(g_{\sigma B}\sigma + g_{\omega B}\omega^\mu\gamma_\mu + g_\rho\vec{R}^\mu\gamma_\mu\vec{\tau}_B\right)\psi_B\end{aligned} \tag{2}$$

$$\mathcal{L}_{YY} = \frac{1}{2}\left(\partial^\mu\sigma^*\partial_\mu\sigma^* - -m_{\sigma^*}^2\sigma^{*2}\right) - \frac{1}{4}S^{\mu\nu}S_{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu - \sum_B \overline{\psi}_B\left(g_{\sigma^* B}\sigma^* + g_{\phi B}\phi^\mu\gamma_\mu\right)\psi_B \tag{3}$$

$$\mathcal{L}_l = \sum_{l=e,\mu} \overline{\psi}\left(i\partial\!\!\!/ - m_l\right)\psi_l \tag{4}$$

In the above equations, ψ_B is the baryon field and the $\sum_B$ runs over all the baryons ($p, n, \Lambda, \Sigma^0, \Sigma^+, \Sigma^-, \Xi^0$ and Ξ^-) and the $\sum_l$ runs over all the leptons. The piece of the Lagrangian $\mathcal{L}_{YY}$ is responsible for the hyperon-hyperon interactions [14]. The meson fields are $\sigma, \omega, \vec{R}(\rho), \sigma^*(f_0(975))$, and ϕ. The U_σ and U_ω are the σ and ω meson potentials [14–16] which are given as :

$$U_\sigma = \frac{b}{3}\sigma^3 + \frac{c}{4}\sigma^4, U_\omega = \frac{d}{4}\omega^4 \tag{5}$$

As mentioned earlier, the TM1 parameter set has been used in this paper. The details of the parameter values may be obtained in ref. [17]. Quark matter EOS is obtained from the standard noninteracting MIT Bag model [18]. Starting from the two models for the hadronic and the quark sectors, a first order deconfinement phase transition is obtained which proceeds via a mixed phase. At zero temperature, in the presence of two conserved charges, the mixed phase is constructed following Gibbs criterion [19]. In the quark sector, we have taken the light quark masses to be zero, the strange quark mass to be 150 MeV and $B^{1/4}$= 180 MeV. The EOS thus obtained has a mixed phase

starting at about $3.8 \times 10^{14} g/cm^3$ and ends at about $1.76 \times 10^{15} g/cm^3$; for the sake of comparison, note that the energy density at nuclear saturation is $2.8 \times 10^{14} g/cm^3$. The corresponding properties of the neutron star as well as the neutron star with a quark core (hybrid star) for the maximum mass configuration are given in Table 1.

TABLE I. Properties of non-rotating and rotating neutron as well as hybrid stars (for maximum mass configuration); Rest mass ($M_0/M_\odot$), gravitational mass ($M/M_\odot$), central energy density (ϵ_c), equatorial radius (R_e) and ratio of polar to equatorial radius (R_p/R_e).

Star motion	Star type	$M_0/M_\odot$	$M/M_\odot$	ϵ_c	R_e	R_p/R_e
Non-rotating	Neutron	1.72	1.57	1.28×10^{15}	13.63	1
	Hybrid	1.53	1.40	3.41×10^{15}	10.20	1
Rotating (Mass shed limit)	Neutron	2.12	1.93	1.10×10^{15}	19.33	0.57
	Hybrid	1.75	1.61	1.79×10^{15}	16.77	0.59

TABLE II. Properties of stars corresponding to the EOS in fig. 1 and fig. 2.

ϵ_c	$M_0/M_\odot$	$M/M_\odot$	Keplarian Frequency of the hybrid star (Hz)	Keplarian Frequency of the initial Neutron Star (Hz)
$6 \times 10^{14} g/cm^3$	1.35	1.27	713	695
$1 \times 10^{15} g/cm^3$	1.62	1.50	895	767

In case of rotating neutron stars, the energy density, and hence the baryon density, profile is substantially different from that of a static star. The density profile can be obtained by solving Einstein's equations using the full EOS. The metric and the procedures involved may be obtained from the ref. [20,21]. The energy density profile of the star, rotating with Keplarian frequency, for two different central densities, $6 \times 10^{14} g/cm^3$ (solid line) and $1 \times 10^{15} g/cm^3$ (dashed line) are shown in figures 1 and 2.

In figure 1, the energy density is plotted against the radial parameter s (integrated over angle $\mu \equiv cos\theta$, θ being the polar angle), which is defined as $R/R_e = s/(1-s)$, R and R_e being the radius and the equatorial radius of the star respectively. Hence, at equator $R = R_e$ so that s=0.5. Figure 1 shows that a higher central energy density results in a sharper variation in the profile.

The μ dependence of energy density (integrated over s) is shown in figure 2. Here, the energy density is found to be much larger towards the polar regions (smaller μ) for higher central energy density, where as, the energy density towards the equator (higher μ) is similar for both the central energy densities. This has a strong bearing on the beaming angle of the emitted neutrinos, as we show later. The properties of the compact star corresponding to the two central energy densities (continuous and dashed curves in figs. 1 and 2) are given in table 2.

Let us now discuss the production of strangeness during the conversion of hadronic matter to SQM. The main reaction mechanism for the production of strange quarks within the quark matter is the non-leptonic weak interaction:

$$u + d \leftrightarrow u + s \tag{6}$$

As discussed earlier, the quark matter initially consists mainly of u and d quarks. Thus, the chemical potentials of u and d are much larger than that of s quark. The process in eqn.(6), converting d to s, releases energy, the amount of which depends on the difference between the d and s chemical potentials. Though reaction (6) is the main agent for s production, the system is driven towards chemical equilibration mainly by the semi-leptonic weak interactions:

$$\begin{aligned} &d(s) \to u + e^- + \bar{\nu}_e; u + e^- \to d(s) + \nu_e; \\ &d(s) + e^+ \to u + \bar{\nu}_e; u \to d(s) + e^+ + \nu_e; \end{aligned} \tag{7}$$

The presence of positrons is mainly important for the cases where due to the trapping of neutrino and energy, the temperature of the reaction region rises. In the present paper, we restrict ourselves to the case of a thin conversion front and assume that there is no trapping of neutrino and energy inside the star. Thus, the star is at a constant temperature. The details of all the cases as well as the behaviour of the reactions are given in ref. [9]. The semi-leptonic reactions are responsible for the neutrino production. For chemically equilibrated matter, the rates of the reactions given by eqn.(7) are much smaller compared to those for non-equilibrated matter [9].

The calculation proceeds as follows. Initial densities of quarks are obtained from the densities of different baryon species in the hadronic matter and their quark content. Final density fractions of the quarks are given by the density profile of the star as given in figures 1 and 2. Transition from the intial to the final state is governed by the rate equations:

$$\begin{aligned} \frac{dn_u(t)}{dt} &= R_{d\to u}(e^-) + R_{s\to u}(e^-) - R_{u\to d}(e^-) - R_{u\to s}(e^-) \\ &+ R_{d\to u}(e^+) + R_{s\to u}(e^+) - R_{u\to d}(e^+) - R_{u\to s}(e^+) \end{aligned} \tag{8}$$

where $R_{d\to u}(e^-)$ is the reaction rate for the u quark production from d quark via electron process. Other rates are defined similarly. One can write down the rate equations for other quarks as well. Solving the coupled rate equations simultaneously along with the chemical equilibrium conditions, we get the neutrino emission rate for different required density fractions. The total number of neutrinos produced during the transition can be obtained by folding this rate with the density profile of the star. Moreover, since the density profile depends on both the radial as well as the polar angles, integrating over radial coordinates gives us the angular distribution of emitted neutrinos. If n_ν is the number of neutrinos emitted per unit time per baryon and n_B is the baryon number density then the number of neutrinos emitted at a particular angle per unit time is given by

$$N_\nu = 2\pi \int r^2 dr\ n_\nu\ n_B \frac{e^{2\alpha+\beta}}{\sqrt{1-v^2}} \tag{9}$$

where α and β are the gravitational potentials and v the rotational velocity. In figure 3 we have plotted the number of neutrinos as a function of μ. One can see that, for central energy density $6 \times 10^{14} g/cm^3$ (solid line), there is a sharp peak between $\mu = 0.1$ and $\mu = 0.24$, the corresponding width being about 12^o. If we increase the central energy density ($1 \times 10^{15} g/cm^3$, dashed line), matter concentration towards the polar regions increases (figure 2) so that the angular variation becomes smaller. This, on the other hand, means that for a star having higher central energy density and hence larger regions of quark matter, the beaming would be less pronounced.

The neutrino beaming found here may be the missing link that causes the Gamma Ray Bursts. In general, the $\nu\bar{\nu} \to e^+e^-$ annihilation cross section is very small. But it has been shown [13] that the general relativistic effects may enhance this cross section substantially and more than 10% of the energy emitted in neutrinos may be deposited in e^+e^- pairs. This enhancement is due to the path bending of the neutrinos which in turn increases the probability of head on $\nu\bar{\nu}$ collision. Though a detailed General Relativistic calculation is needed to quantify the resulting increment due to beaming, one can safely infer that beaming would increase the efficiency of the $\nu\bar{\nu} \to e^+e^-$ process further, providing a very efficient engine for the Gamma Ray Bursts.

Our estimate shows that about 10^{52} ergs of energy is released in the form of neutrinos, the average energy of each neutrino being of the order of 100 MeV. But some of the neutrinos may get trapped in the interior, thereby producing more $\nu\bar{\nu}$ pairs. So the final number of neutrinos may somewhat be larger than the present estimate. Moreover, the trapping of neutrinos, and consequently energy, would result in an increase in temperature on the stellar interior. This, in turn, would result in further enhancement of the $\nu\bar{\nu} \to e^+e^-$ cross section due to the gravitational red shift effect [13].

Since the time required for the conversion of hadronic matter to quark matter at each density is calculable, one can estimate the time scale for the conversion of the neutron star to hybrid star or quark star. It has been shown earlier [9] that for a fixed temperature (say, T=10 MeV) the time taken to form the chemically equilibrated quark matter at density 0.6 fm^{-3} is around 0.1 s. This time would be smaller for lower density and/or higher temperature. In the present case the density at the core varies between 0.30 - 0.52 fm^{-3} and the corresponding time scale for chemical equilibration is 10^{-3} - 10^{-1}. Since Neutrinos are being emitted throughout during the conversion, this time scale would roughly be equal to the time duration of the Gamma Ray Burst. Hence, in our model, the duration of the Gamma Ray Burst is expected to be of the order of 10^{-3} - 10^{-1} seconds. Observationally, the duration of GRBs, range from 10^{-3} sec. to about 10^3 sec. with a well defined bimodal distribution for bursts longer (long & soft GRBs) or shorter (short & hard GRBs) than t $\approx$ 2 Sec. [22,23]. So the present scenario would act as an engine for short & hard GRBs.

To conclude, we have shown that the rotation of the neutron star causes a beaming of the neutrinos produced during the hadron to quark phase transition inside the star. This beaming, along with the general relativistic effects, can substantially enhance the neutrino annihilation cross section and thus provide a very efficient engine for Gamma Ray Bursts. The beaming angle depends on the extent of the quark phase inside the star. If the quark phase extent is smaller, then the neutrinos are emitted within a narrower angle and the corresponding time duration of the burst would be smaller. For the parameter values used here, the calculated angle, the emitted energy as well as the duration of the burst compare quite well with the observed values.

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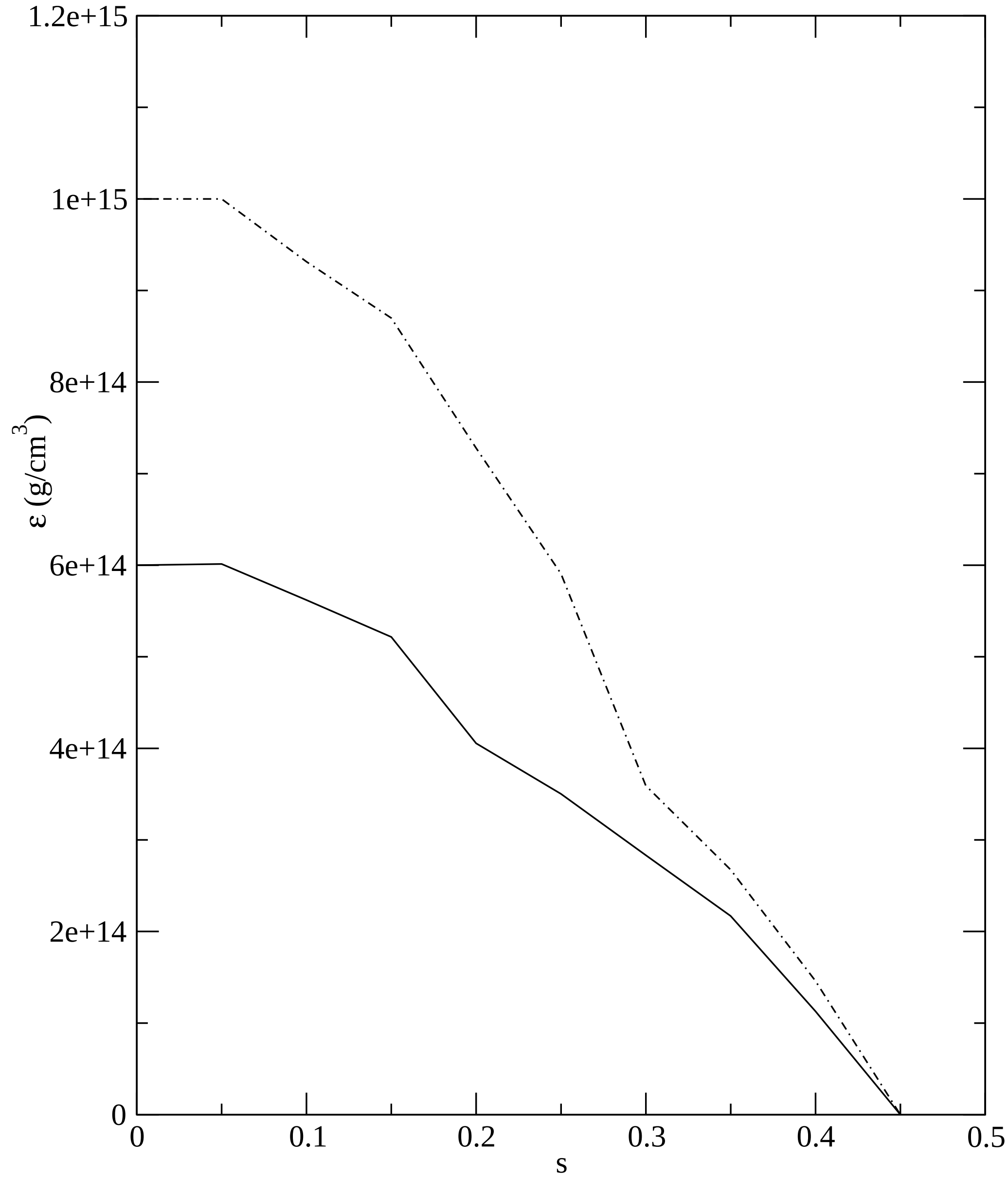

FIG. 1. Energy Density variation (integrated over μ) with radial parameter s of the star for two central energy densities ; $6 \times 10^{14} g/cm^3$ (solid line) and $1 \times 10^{15} g/cm^3$ (dashed line)

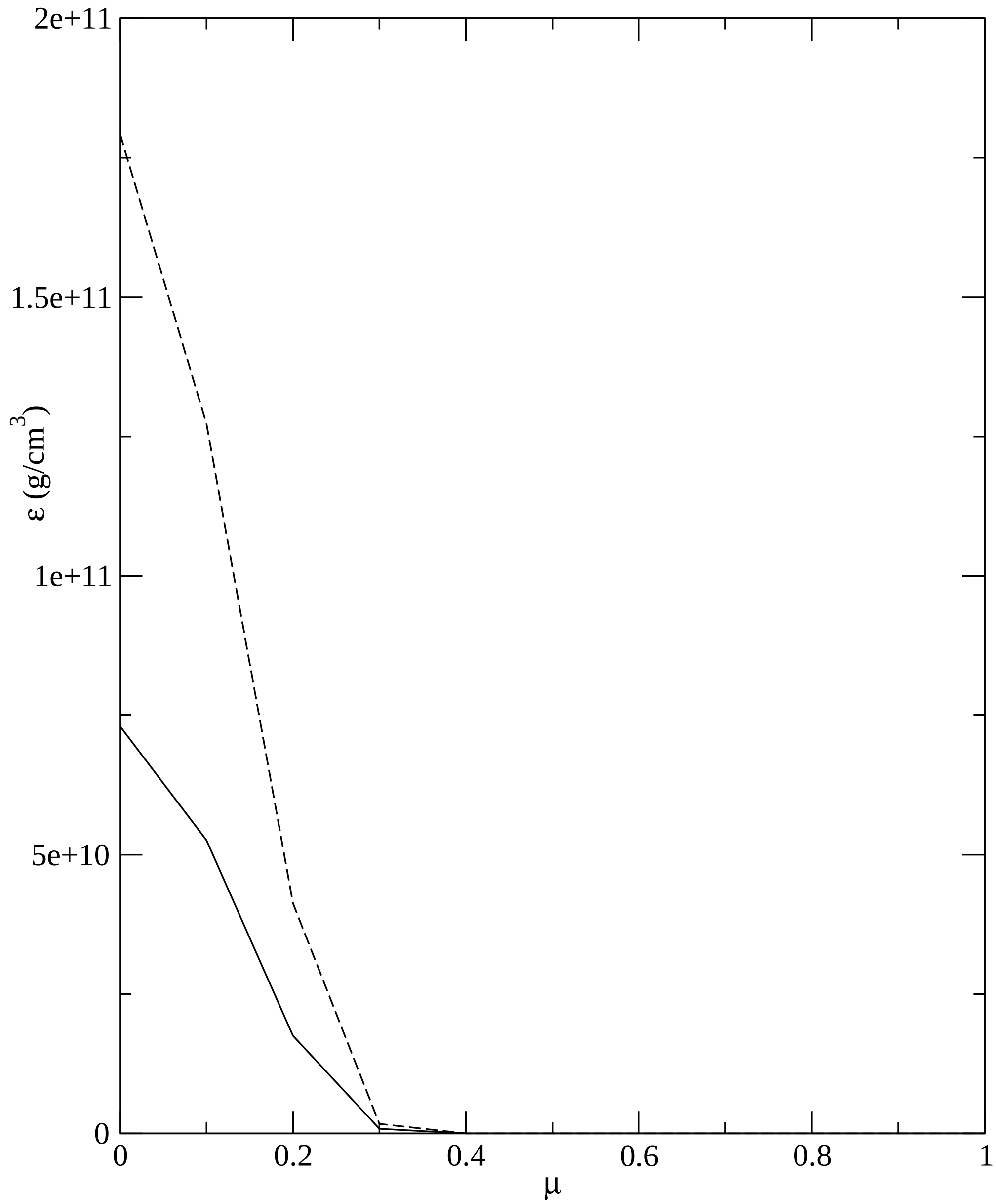

FIG. 2. Angular distribution of the energy Density (integrated over radial parameter s) of the star for two central energy densities ; $6 \times 10^{14} g/cm^3$ (solid line) and $1 \times 10^{15} g/cm^3$ (dashed line)

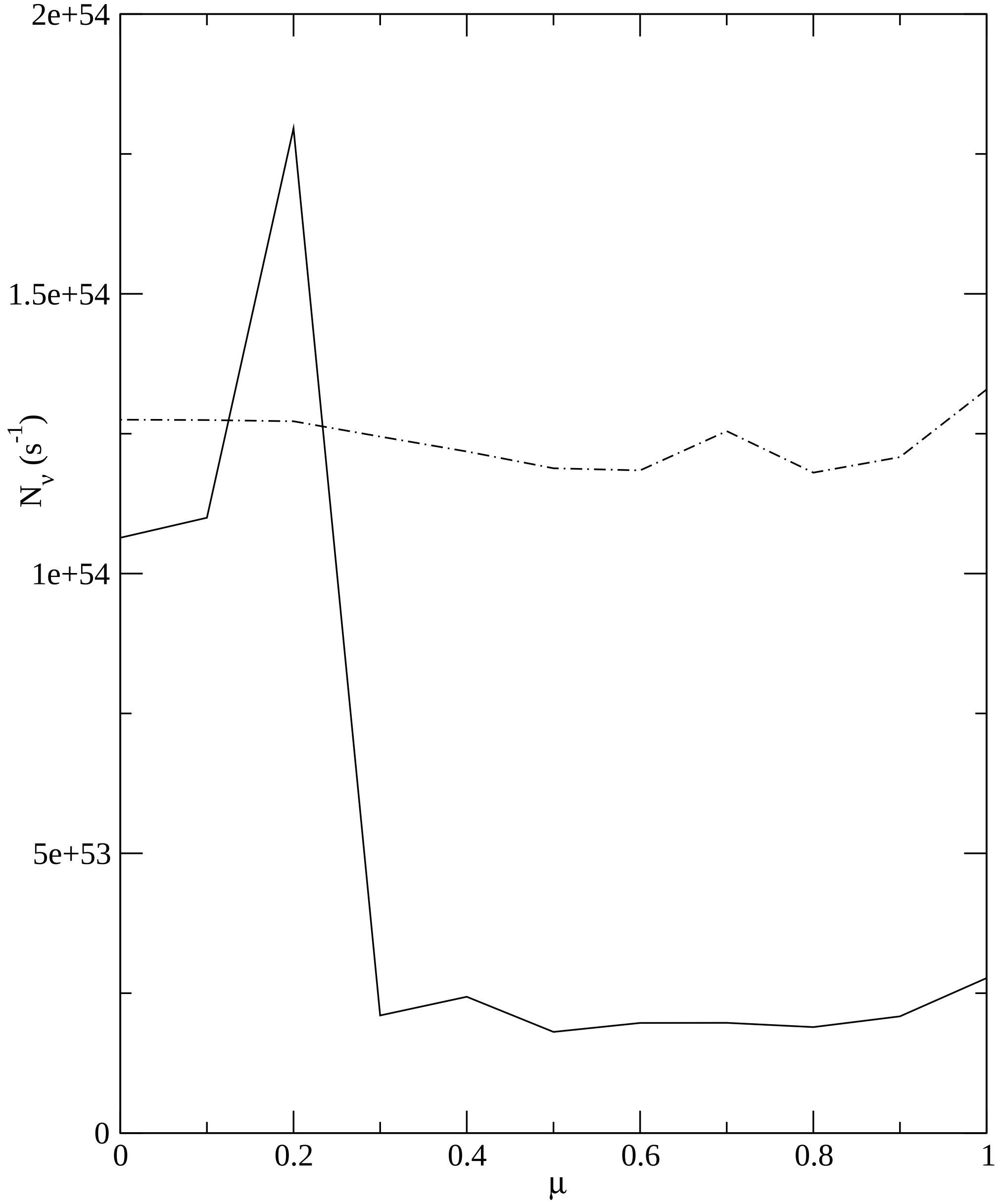

FIG. 3. Neutrino emission as a function of μ for the central energy densities as in figs. 1 and 2.